

# Theoretical Considerations in the Zone Melting of Organic Compounds. Segregation of Impurities

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A new concept is presented, along with its mathematical analysis, to the zone melting technique as applied to organic pharmaceutical compounds. Its purpose is to delineate a unique property inherent in most organics which is essential in considering the zone melting theory. The use of this concept, based on the analysis of eutectic forming systems, reaches into the more general area of physical-organic properties. The quantitative treatment presented offers a new means of characterizing organic substances independent of their chemical nature.

AN EXPLANATION of the principles involved in the zone melting process for organic compounds may be analyzed in two parts: (a) the tendency of the major component to freeze out pure at the freezing interface, and (b) the transport process of the impurity as it is distributed throughout the molten zone.

References are available for the role played by the transport process in preceding papers (1). Attention will now be focused on the freezing interface of a eutectic mixture under ideal equilibrium conditions, where the major component tends to freeze out pure, leaving a narrow band of impurity at the freezing interface in the molten zone. The question of why the impurity does not freeze out with the major component arises. The thermodynamic equations describing the lowering of the freezing point by impurities give no evidence to either effect (2). Some textbooks (3) imply that it is not possible to show by thermodynamic means that the impurity will remain in the liquid melt. Burton (4) raises the question of the chemical exchange at the interface in the zone melting of metals but offers no interpretation.

The point of view taken here<sup>1</sup> is that, in the case of pure diffusion, the equations describing the transport of impurity through the melted zone are related to the force necessary to prevent the impurities from crystallizing with the freezing front. In other words, from the equations of diffusion describing the transport process, the diffusive force which originates at the freezing boundary can be calculated. This diffusive force represents a measure of the purification process occurring at the freezing interface. For those less familiar with zone melting literature, it may be pointed out that all substances tend to form either eutectic or solution type mixtures. There is no question that, in the case where a substance forms a solid solution and is zone melted, the process is comparable to a solution or distillation phenomena. However, most organics form eutectic type mixtures, the most common example of which is water crystallizing out 100% pure from salt solutions. The question of why a partition coefficient

is not necessary for such types of mixtures arises. Thus, the partition coefficient is a function of the phase diagram only in the case of solid solutions. It is true that one can have a partition coefficient in connection with a eutectic mixture, e.g., a salt solution, if the freezing front moves rapidly (since it will trap or occlude the impurity or salt). But this partitioning is a function of the *conditions* and not a function of the *phase diagram*. Therefore, under *ideal equilibrium conditions* with a eutectic, one has no occlusion, and the major component freezes out pure. The significance of this phenomenon raises the fundamental question of why do organics form eutectics and metals and inorganics form solid solutions. The thermodynamic equations for lowering of the freezing point by impurities (freezing point depression equations) are identical for the two kinds of materials. In other words, the freezing point depression equations offer no way of determining whether a substance will form a eutectic or solid solution. Most scientists simply accept the fact that there are two different types of phase diagrams; in this article it is questioned why this should be. Assuming that there are two different kinds of properties of matter involved, for the first time a mathematical method is proposed for distinguishing between them.

## DIFFUSIVE FORCE

Diffusion is defined as the migration of a substance under the action of a difference in chemical potential. In other words, the diffusive force equals the negative gradient of chemical potential

$$f_i = - \frac{1}{N} \left( \frac{\partial \mu_i}{\partial x} \right)_T \quad (\text{Eq. 1})$$

where  $f_i$  = the force on a single particle,  $N$  = Avogadro's number, and  $i$  indicates a single particle.

For sufficiently dilute solutions

$$\mu_i = RT \ln C_i + \mu_i^\circ \quad (\text{Eq. 2})$$

$$\frac{\partial \mu_i}{\partial x} = \frac{RT}{C_i} \frac{\partial C_i}{\partial x} \quad (\text{Eq. 3})$$

then

$$-Nf_i = \frac{\partial \mu}{\partial x} = \frac{RT}{C} \frac{\partial C}{\partial x} \quad (\text{Eq. 4})$$

The velocity under the action of unit force is given by the generalized mobility,  $V_i$ , so that

$$V = - \frac{V_i}{N} \frac{\partial \mu_i}{\partial x} \quad (\text{Eq. 5})$$

The net flow of material through unit cross section

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in unit time is therefore

$$\frac{\partial Q}{\partial t} = - \frac{V_i n_i}{N} \frac{\partial \mu_i}{\partial x} \quad (\text{Eq. 6})$$

where  $n_i$  = number of particles in unit volume.

Substituting Eqs. 4 and 5 into Eq. 6 yields

$$Nf_i = - \frac{\partial \mu}{\partial x} = - \frac{\partial Q}{\partial t} \frac{N}{n_i} \frac{1}{V_i} = \frac{RT}{C} \frac{\partial C}{\partial x} \quad (\text{Eq. 7})$$

The diffusion coefficient,  $D_i = KTV_i$ , where  $R/N = K$ . Rearranging Fick's first law of diffusion and substituting for  $D_i$  yields, on a molar basis,

$$Nf_i = - \frac{\partial \mu}{\partial x} = \frac{\partial Q}{\partial t} = -D_i \frac{\partial C}{\partial x} \quad (\text{Eq. 8})$$

which relates the diffusive force to the negative gradient of chemical potential to Fick's first law of diffusion.

The force,  $f$ , on the number of particles in concentration,  $C$ , then is given by

$$f = - \left( \frac{\partial \mu}{\partial x} \right)_T = \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad (\text{Eq. 9})$$

which was derived in an earlier work (5). The solution to this equation is given also in the preceding work, such that for the condition that  $(VL/D) > 3.0$  or  $< 2.0$  as a first approximation at the freezing interface.

$$f = - \frac{C_0 V}{D} [2 + (V^2 t/D)] \quad (\text{Eq. 10})$$

The condition that  $(VL/D) > 3.0$  is reasonable for this zone melting process where the order of magnitudes are  $V \sim 10^{-6}$  cm. sec.<sup>-1</sup> or less,  $D \sim 1.0$  to  $3.5 \times 10^{-8}$ , and  $L \geq 2.0$  cm.

Equation 10 may be considered a measure of the forces acting at the freezing interface, preventing refreezing of the impurity with the major component. Examination of this equation indicates that the forces acting at the freezing interface to prevent resolidification of impurity are dependent upon  $C_0$ , the initial concentration,  $V$ , the velocity of the zone movement,  $D$ , the diffusion coefficient, and  $t$ , the time. The forces are independent of the length of the zone; although for this particular solution,  $L$  is of the order of magnitude of 3.0 cm. or more, due to

the particular method used in arriving at the solution.

## DISCUSSION

Wilcox (6) also describes the differential equation and boundary conditions governing pure diffusion in zone melting. The work, done independently and concurrently, is slightly different in form from that presented under *Diffusive Force* but is essentially the same in principle. However, Wilcox did not arrive at an analytical solution to the diffusion equation, nor was he concerned with arriving at a measure of the diffusive force. His purpose was merely to describe the transport process for diffusion in zone melting as a means of expanding knowledge.

The solution to the differential Eq. 8 to yield Eq. 10 for the specific conditions imposed by the zone melting process of an individual compound will yield a quantitative measure of the force involved in the segregation of the impurities into the liquid melt of that compound. The full application of this equation to specific substances during the zone melting process can be the subject of future theoretical endeavor. The significance and ramifications of the specific solutions of this equation to ideally zone melted materials may shed new light on a fundamental property of organic compounds. The theoretical basis of the 100% movement of impurities into the melt zone at the freezing interface is not understood at all. The evidence of 100% movement of impurity suggests itself to be contrary to the normal experiences of solution and distillation phenomena which are continuous fractional processes. The total movement of impurity, represented by the diffusive force, signifies a complete discontinuous process, an indication that a different property of matter is involved. The full explanation of this unique behavior of eutectic materials offers a stimulating challenge to the theoretical chemist.

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